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COSMOLOGY: FROM HUBBLE TO HST

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The Hubble constant sets the size and age of the Universe, and, together with independent determinations of the age, provides a consistency check of the standard cosmology. The Hubble constant also provides an important test of our most attractive paradigm for extending the standard cosmology, inflation and cold dark matter.

1   Introduction

The value of the Hubble constant has changed by about a factor of ten since Edwin Hubble's pioneering measurements. The context in which we view the Universe has changed just as profoundly. Until 1964 cosmology was mostly concerned with cosmography; the spirit of this period was perhaps best captured by Sandage, "the quest for two numbers (H_0 and q_0)." The discovery of the Cosmic Background Radiation led to the establishment of a physical foundation for the expanding Universe – the hot big-bang cosmology. In the 1970s saw this model become firmly established as the standard cosmology. In the 1980s cosmologists began trying to extend the standard cosmology by rooting it in fundamental physics. Inflation is the first step in this program. Today, a host of cosmological observations are testing inflation and its cold dark matter theory of structure formation; here I focus on the role that the Hubble constant is playing in this enterprise.

2   Foundations

The hot big-bang cosmology is a remarkable achievement. It provides a reliable accounting of the Universe from around 10^{-2} sec until the present, some 10 Gyr to 15 Gyr later. It, together with the standard model of particle physics and speculations about the unification of the fundamental forces and particles, provides a firm foundation for the sensible discussion of earlier times.

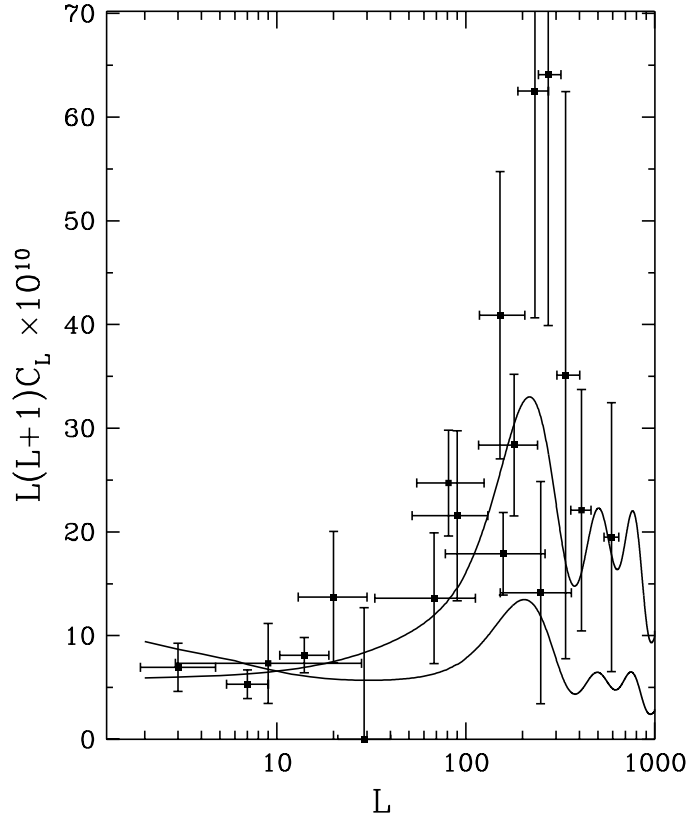The standard cosmology rests on four observational pillars:

Figure 1: Summary of CBR anisotropy measurements. Plotted are the squares of the measured multipole amplitudes ($C_l = \langle |a_{lm}|^2 \rangle$) versus multipole number l. The relative temperature difference on angular scale θ is given roughly by $\sqrt{l(l+1)C_l/2\pi}$ with $l \sim 200°/\theta$. The theoretical curves are standard CDM (upper curve) and CDM with $n = 0.7$ and $h = 0.5$ (lower curve).

- The expansion of the Universe. The redshifts and distances of thousands of galaxies have been measured and are in accord with Hubble's Law, $z = H_0 d$, a prediction of big-bang models for $z \ll 1$.

- The Cosmic Background Radiation (CBR). The CBR is the most precise black body known – deviations from the Planck law are smaller than 0.03% of the maximum intensity. Its temperature has been measured to four significant figures: $T_0 = 2.728 \pm 0.002\,\mathrm{K}$ [1]. The only plausible origin is the hot, dense plasma that existed in the Universe at times earlier than $10^{13}\,\mathrm{sec}$ (epoch of last scattering and recombination).

- Temperature fluctuations in the CBR. Temperature differences of order $30\mu\mathrm{K}$ between directions on the sky separated by angles from less than one degree to ninety degrees have been measured by more than ten different experiments [2] (Fig. 1). They establish the existence of density inhomogeneities at the same level, $\delta\rho/\rho \sim \delta T/T \sim 10^{-5}$, on
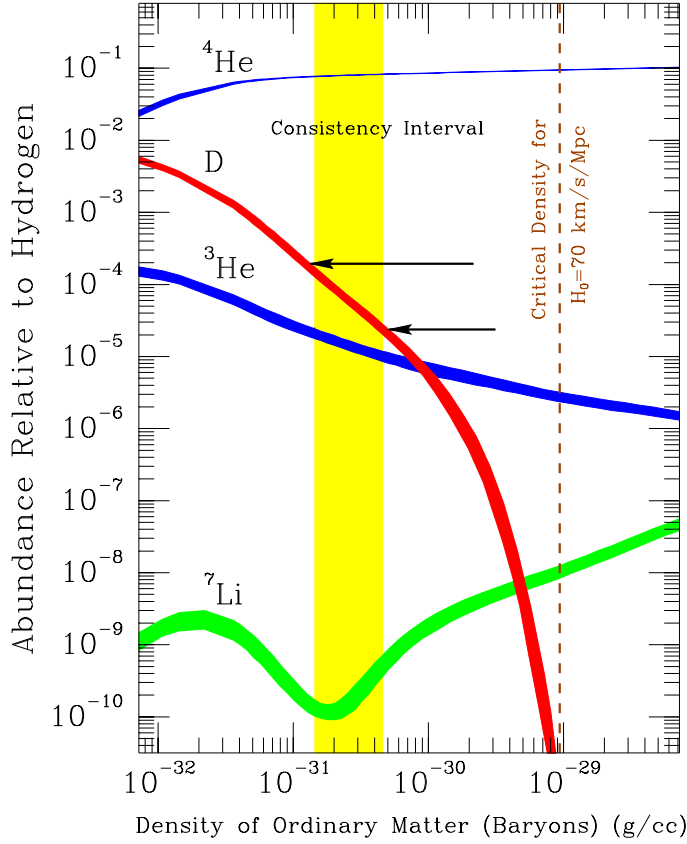
Figure 2: Big-bang production of the light elements; widths of the curves show the two-sigma theoretical uncertainty. The primeval abundances of D, ^{3}He, ^{4}He and ^{7}Li can be explained if the baryon density is between 1.5×10^{-31} g cm^{-3} and 4.5×10^{-31} g cm^{-3} ($\Omega_B h^2 = 0.008 - 0.024$).

length scales $\lambda \sim 100 h^{-1}$ Mpc $(\theta/\text{deg}) \sim 30 h^{-1}$ Mpc $- 10^4 h^{-1}$ Mpc. Density perturbations of this amplitude, when amplified by the attractive action of gravity over the age of the Universe, are sufficient to explain the structure seen today.

- Primeval abundance pattern of D, ^{3}He, ^{4}He and ^{7}Li. These light nuclei were produced a few seconds after the bang; the predicted abundance pattern is consistent that seen in primitive samples of the cosmos – provided that the present baryon density is between 1.5×10^{-31} g cm^{-3} and 4.5×10^{-31} g cm^{-3}. This corresponds to a fraction of critical density $\Omega_B h^2 = 0.008 - 0.024$ [3] (Fig. 2). Nucleosynthesis is the earliest test of the hot big bang and provides the best determination of the density of ordinary matter.

The standard cosmology is successful in spite of our ignorance of the basic geometry of the Universe – age, size, and curvature – which hinge upon accurate measurements of the Hubble constant and energy content of the Universe (fraction of critical density in matter, radiation, vacuum energy, and so on). The expansion age, which is related to H_0^{-1} and the

energy content of the Universe, is an important consistency check – it should be larger than the age of any object in the Universe. The curvature radius of the Universe is related to H_0 and Ω_0: $R_{\mathrm{curv}} = H_0^{-1}/\sqrt{|\Omega_0 - 1|}$.

Note, the deceleration parameter is related to energy content of the Universe, $q_0 = \frac{1}{2}(\Omega_0 + 3\sum_i w_i \Omega_i)$, where Ω_0 is the total energy density divided the critical energy density, Ω_i is the fraction of critical density in component i and w_i is the ratio of the pressure contributed by component i to its energy density. For a universe filled with nonrelativistic matter, $q_0 = \frac{1}{2}\Omega_0$; for a universe with nonrelativistic matter + vacuum energy (cosmological constant, $w_\Lambda = -1$), $q_0 = \frac{1}{2}\Omega_0 - \frac{3}{2}\Omega_\Lambda$.

3 Aspirations

The hot big-bang model provides a firm physical basis for the expanding Universe, but it leaves important questions unanswered.

- Quantity and composition of dark matter. Most of the matter in the Universe is dark and of unknown composition [4]. The peculiar velocities of the Milky Way and other galaxies indicate that Ω_{Matter} is at least 0.3, perhaps as large as unity [5]. Luminous matter accounts for less mass density that the lower limit to the baryon density from nucleosynthesis ($\Omega_{\mathrm{Lum}} \simeq 0.003h^{-1} < 0.008h^{-2} < \Omega_B$), and the upper limit to the baryon density from nucleosynthesis is less than 0.3 ($\Omega_B < 0.024h^{-2} < 0.3$). This defines the two dark-matter problems central to cosmology (Fig. 3). What is the nature of the dark baryons? What is the nature of the nonbaryonic dark matter?

- Formation of large-scale structure. Gravitational amplification of small primeval density inhomogeneities provides the basic framework for understanding structure formation, but important questions remain. What is the origin of these perturbations? In detail, how did structure evolve? The latter is clearly tied to the dark-matter question.

- Origin of matter-antimatter asymmetry. During the earliest moments ($t \lesssim 10^{-6}\,\mathrm{sec}$), when temperatures exceeded the rest-mass energy of nucleons, matter and antimatter existed in almost equal amounts (thermal pair production made nucleons and antinucleons as abundant as photons); today there is no antimatter and relatively little matter (one atom for every billion photons). For this to be so, there must have been a slight excess of matter over antimatter during the earliest moments: about one extra nucleon per billion nucleons and antinucleons, for a net baryon number per photon of about 10^{-9}. What is the origin of this small baryon number?

- Origin of smoothness and flatness. Why in the large is the Universe so smooth (as evidenced by the CBR)? The generic cosmological solutions to Einstein's equations are not smooth; further, microphysical processes could not have smoothed things out because the distance a light signal can travel at early times covers only a small fraction of the Universe we can see. Why was the Universe so flat in the beginning? Had it not
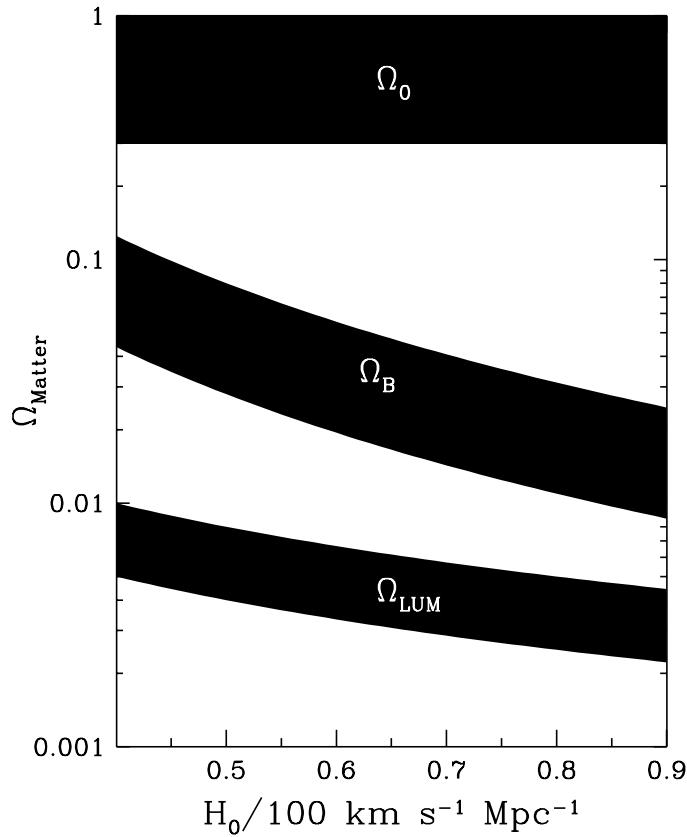
Figure 3: Determinations of the matter density. The lowest band is luminous matter, in the form of bright stars and associated material; the middle band is the big-bang nucleosynthesis determination of the density of baryons; the upper region is the estimate of Ω_{Matter} based upon the peculiar velocities of galaxies. The gaps between the bands illustrate the two dark matter problems: most of the ordinary matter is dark and most of the matter is nonbaryonic.

been exceedingly flat, it would have long ago recollapsed or gone into free expansion, resulting in a CBR temperature of much less than 3 K.

- The beginning. What launched the expansion? What is the origin of the entropy (i.e., CBR)? What was the big bang? Is there a before the big bang? Were there other bangs? Are there more spatial dimensions to be discovered?

This is an ambitious list of questions. If physical explanations can be found, we will have a more fundamental understanding of the Universe. The study of the unification of the forces of Nature and the application of these ideas to the early Universe has allowed these questions to be addressed, and many of us believe that answers will be found in the physics of the early Universe. Over the past fifteen years a number of important ideas have been put forth [6] – baryogenesis, topological defects (cosmic strings, monopoles, textures, and domain walls), particle dark matter, baryogenesis, and inflation. I will focus on inflation –

it is the most expansive, addresses almost all the questions mentioned above, and is ripe for testing.

4   Inflation and Cold Dark Matter

Inflation [7] holds that very early on (perhaps around 10^{-34} sec) the Universe underwent a burst of exponential expansion driven by the energy of a scalar field displaced from the minimum of its potential-energy curve. (There are many candidates for the scalar field that drives inflation; all involve new fields associated with physics beyond the standard model of particle physics.) During this growth spurt, the Universe expanded by a larger factor than it has since. Eventually the scalar field evolved to the minimum of its potential and its energy was released into a thermal bath of particles. This entropy is still with us today: the Cosmic Background Radiation.

The tremendous growth in size during inflation explains the large-scale flatness and smoothness of the Universe: After inflation, a very tiny patch of the pre-inflationary Universe, which would necessarily appear flat and smooth, becomes large enough to encompass all that we see today and more. Since spatial curvature and Ω_0 are related, inflation predicts a critical density Universe.[1]

The most stunning prediction of inflation is the linking of large-scale structure in the Universe to quantum fluctuations on microscopic scales [9] ($\ll 10^{-16}$ cm): The wavelengths of quantum fluctuations in the scalar field that drives inflation are stretched to astrophysical size by the expansion that occurs during inflation. The continual creation of quantum fluctuations and expansion leads to fluctuations on all length scales; they develop into density perturbations when the vacuum energy is converted into radiation. The spectrum is approximately scale invariant, that is, fluctuations in the gravitational potential that are independent of length scale. The overall normalization of the spectrum is dependent upon the shape of the scalar potential, and achieving fluctuations of the correct size to produce the observed structure in the Universe places an important constraint on it.

An inflationary model must incorporate two other pieces of early-Universe physics: baryogenesis [10] and particle dark matter [11]. Since the massive entropy released at the end of inflation exponentially dilutes any asymmetry that might have existed between matter and antimatter, an explanation for the matter – antimatter asymmetry must be provided. Baryogenesis is an attractive one. It holds that particle interactions that do not conserve baryon-number and do not respect C and CP (matter-antimatter) symmetry occurred out-of-thermal-equilibrium and gave rise to the small excess of matter over antimatter needed to ensure the existence of matter today. Details of baryogenesis remain to be worked out and tested – did baryogenesis occur at modest temperatures $T \sim 200$ GeV and involve the baryon-number violation that exists in the standard model or did it occur at much higher temperatures and involve grand unification physics.

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[1]Recently, it has been shown that inflation can accommodate $\Omega_0 < 1$, but at the expense of tuning precisely the amount of inflation [8].

Particle dark matter is necessary since inflation predicts that the Universe is at the critical density and baryons can contribute at most 10% of that. While the standard model of particle physics does not provide a particle dark matter candidate, many theories that attempt to unify the forces and particles predict the existence of new, long-lived particles whose abundance today is sufficient to provide the critical mass density. The three most promising candidates are: a neutrino of mass around 30 eV; a neutralino of mass between 10 GeV and 500 GeV [12]; and an axion of mass between 10^{-6} eV and 10^{-4} eV [13].

Inflation addresses essentially all the previously mentioned questions, including the nature of the big bang itself. As Linde [14] has emphasized, if inflation occurred, it has occurred time and time again (eternally to use Linde's words). What we refer to as the big bang is simply the beginning of our inflationary bubble, one of an infinite number that have been spawned and will continue to be spawned ad infinitum. From the inflationary view, there is no need for a beginning. (In that way, inflation is similar to steady-state cosmology.)

There is no standard model of inflation, but there are a set of robust predictions that allow inflation to be tested.

- Flat Universe. Total energy density is equal to the critical density, $\sum_i \Omega_i = 1$. Among the components i are baryons, slowly moving elementary particles (cold dark matter), radiation (a very minor component today, $\Omega_{\rm rad} \sim 10^{-4}$), and possibly other particle relics or a cosmological constant.

- Approximately scale-invariant spectrum of density perturbations. More precisely, the Fourier components of the primeval density field are drawn from a gaussian distribution with variance given by power spectrum $P(k) \equiv \langle |\delta_k|^2 \rangle = Ak^n$ with $n \approx 1$ ($n = 1$ is exact scale invariance), where $k = 2\pi/\lambda$ is wavenumber and the model-dependent constant A sets the overall level of inhomogeneity and is related to the form of the inflationary potential.

- Approximately scale-invariant spectrum of gravitational waves. Quantum fluctuations in the space-time metric give rise to relic gravitational waves. The overall amplitude of the spectrum depends upon the scalar potential in a different way than the density perturbations. These relic gravitational waves might be detected directly by laser interferometers that are being built (LIGO, VIRGO, and LISA) or by the CBR anisotropies they produce [15]. If the spectra of both the matter fluctuations and gravity waves can be determined, much could be learned about the inflationary potential [16].

The first two predictions lead to the cold dark matter (CDM) theory of structure formation.[2] Within the cold dark matter theory, there are cosmological quantities that must be

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[2]As a historical note the more conservative approach of neutrino (hot) dark matter was tried first and found to be wanting [17]: Since neutrinos are light and move very fast they stream out of overdense regions and into underdense regions, smoothing out density inhomogeneities on small scales. Structure forms from the top down: superclusters fragmenting into galaxies – which is inconsistent with observations that indicate that superclusters are just forming today and galaxies formed long ago.

specified in order to make precise predictions [20]. They can be organized into two groups. First are the cosmological parameters: the Hubble constant; the density of ordinary matter; the power-law index n and overall normalization constant A that quantify the density perturbations; and the level of gravitational radiation.[3] (A given model of inflation predicts A and n as well as the level of gravitational radiation; however, there is no standard model of inflation. Conversely, measurements of the above quantities can constrain – and even be used to reconstruct – the scalar potential that drives inflation [16].)

The second group specifies the composition of invisible matter in the Universe: radiation, dark matter, and cosmological constant. Radiation refers to relativistic particles: the photons in the CBR, three massless neutrino species (assuming none of the neutrino species has a mass), and possibly other undetected relativistic particles. The level of radiation is crucial since it determines when the growth of structure begins and thereby the shape of the power spectrum of density perturbations today. While the bulk of the dark matter is CDM, there could be other particle relics; for example, a neutrino species of mass $5\,\mathrm{eV}$, which would account for about 20% of the critical density.

The testing of cold dark matter began more than a decade ago with a default set of parameters ("standard CDM") characterized by simple choices for both the cosmological and the invisible matter parameters: precisely scale-invariant density perturbations ($n = 1$), $h = 0.5, \Omega_B = 0.05, \Omega_{\mathrm{CDM}} = 0.95$; no radiation beyond photons and three massless neutrinos; no dark matter beyond CDM; no gravitational waves; and zero cosmological constant. The overall level of the matter inhomogeneity – set by the constant A – was fixed by comparing the predicted level of inhomogeneity today with that seen in the distribution of bright galaxies. Bright galaxies may or may not faithfully trace the distribution of mass. In fact, there is some evidence that bright galaxies are more clustered than mass, by a factor called the bias, $b \simeq 1 - 2$. The distribution of galaxies today only fixes A up to the bias factor b.

An important change occurred with the detection of CBR anisotropy by COBE in 1992 [21]. The COBE measurement permitted a precise determination of the amplitude of density perturbations on very large scales, without regard to biasing. And there was a surprise: For standard CDM, the COBE normalization predicts too much power on the scales of clusters and smaller [19].

Figure 4 illustrates clearly that this problem simply reflects a poor choice for the standard parameters. It shows that there are many COBE-normalized CDM models that are consistent with measurements of the large-scale structure that exists today (shape of the power spectrum of the galaxy distribution, abundance of clusters, and early formation of structure in the form of damped Lyman-α clouds; see Ref. [20]). Organized into families characterized by their invisible matter content they are: CDM + cosmological constant (ΛCDM) [22], CDM + a small amount of hot dark matter (νCDM) [23], CDM + additional relativistic particles (τCDM) [24], and CDM with standard invisible matter content [25, 26].

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[3]The level of gravitational radiation is important because density perturbations are normalized by CBR anisotropy and at present it is difficult to separate the contribution of gravity waves to CBR anisotropy from that due to density perturbations [18].
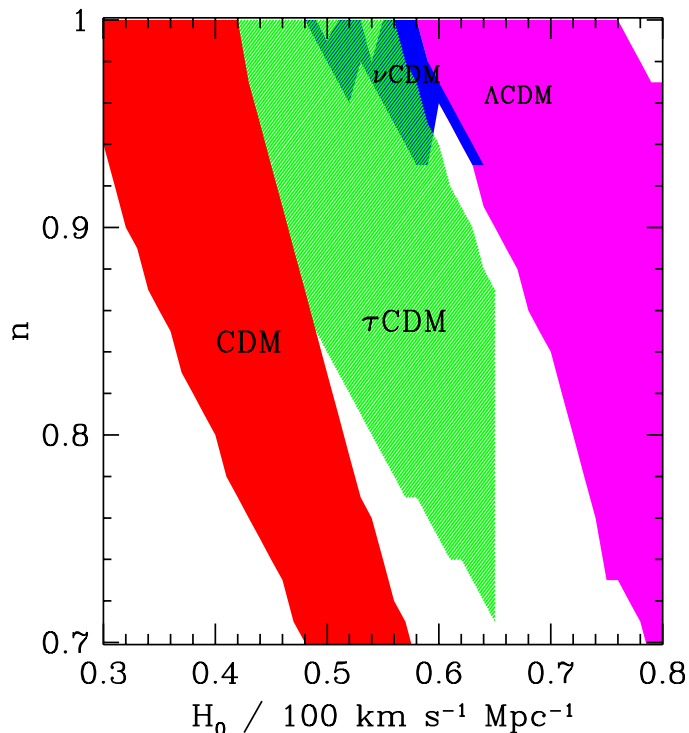
Figure 4: Acceptable values of the cosmological parameters n and h for CDM models with standard invisible-matter content (CDM), with 20% hot dark matter (νCDM), with additional relativistic particles (the energy equivalent of 12 massless neutrino species, denoted τCDM), and with a cosmological constant that accounts for 60% of the critical density (ΛCDM). The τCDM models have been truncated at a Hubble constant of $65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ because a larger value would result in a Universe that is younger than 10 Gyr (from Ref. [20]).

5 H_0 Tests Inflation and Cold Dark Matter

A flood of cosmological observations – from determinations of the Hubble constant to measurements of CBR anisotropy – are now sharply testing inflation and cold dark matter. Here I will focus on the important role that H_0 plays. It is two fold: age-Hubble constant consistency and shape of the power spectrum of inhomogeneity today (which depends upon H_0 as it determines the value of the critical density and thereby the epoch of matter-radiation equality).

The determinations of the ages of the oldest stars lie between 12 Gyr and 17 Gyr [27, 28]. These estimates recent support from two other independent methods – the dating of the oldest white dwarfs based upon how they cool and the dating of the radioactive elements, e.g., the isotope ratio of $^{235}\mathrm{U}/^{238}\mathrm{U}$ [29]. Taken together, the case for an absolute minimum age of 10 Gyr appears ironclad.

On the other hand, measurements of the Hubble constant now favor values between $60\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $80\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, which for $\Omega_{\mathrm{Matter}} = 1$ implies an expansion age of

11 Gyr or less. For a flat Universe with a cosmological constant the expansion age is greater than $\frac{2}{3} H_0^{-1}$, which lessens the age problem. Within the uncertainties there is no inconsistency, though there is tension, especially for models with $\Omega_{\text{Matter}} = 1$ (Fig. 5). Large-scale structure considerations ease the age problem, as they favor an older Universe by virtue of a lower Hubble constant or cosmological constant (Fig. 4). Still, the Hubble constant has great leverage. Consider the following:

- $H_0 < 60 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$. CDM with standard invisible matter content is viable. However, the closer H_0 is to $60 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$, the more tilt (deviation of n from unity) is required. CBR anisotropy precludes n less than 0.7; the next generation of satellite experiments, MAP and COBRAS/SAMBA, should be able to determine n to an accuracy of a few percent.

- $60 \, \text{km s}^{-1} \, \text{Mpc}^{-1} < H_0 < 65 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$. Only models with nonstandard invisible-matter content are viable, e.g., νCDM and τCDM. νCDM has a smokin' gun signature: around $5 \, \text{eV}$ worth of neutrino mass (in one or more species). Particle-physics models for producing extra relativistic particles (τCDM) call for a massive ($1 \, \text{keV} - 10 \, \text{MeV}$), unstable tau neutrino. There are a host of laboratory experiments searching for evidence of neutrino mass.

- $H_0 > 65 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$. Only ΛCDM is viable. ΛCDM too has a smokin' gun signature: $q_0 = \frac{1}{2} - \frac{3}{2} \Omega_\Lambda \approx -0.5$. This should be tested soon by the two groups using distant ($z \sim 0.3 - 0.7$) Type Ia supernovae to measure q_0.

Another test of inflation and CDM involves H_0, though less directly. Because clusters of galaxies are large objects it is expected that the cluster baryon fraction, determined from x-ray measurements to be $(0.04 - 0.10) h^{-3/2}$ [30], should closely reflect its universal value, $\Omega_B / \Omega_{\text{Matter}}$. Using the nucleosynthesis value for Ω_B fixes Ω_{Matter} to be $(0.1 - 0.6) h^{-1/2}$. Unless H_0 is very low, this determination of Ω_{Matter} is only consistent with ΛCDM. However, it should be remembered that important assumptions must be made to infer the cluster baryon fraction – that the hot intracluster gas is unclumped and supported by thermal pressure alone – if either is untrue the actual baryon fraction would be smaller.

6 Concluding Remarks

This is an exciting time in a cosmology. We have a very successful standard model, the hot big-bang cosmology, a bold and expansive paradigm for extending it, inflation and cold dark matter, and the observations that can test it are flooding in. As I have emphasized, the value of the Hubble constant has an important role in this enterprise. And of course, the Hubble constant provides a consistency check of the standard cosmology.

As we have heard at this meeting great progress is being made. The Hubble Space Telescope is obtaining accurate Cepheid distances to galaxies which can be used to calibrate
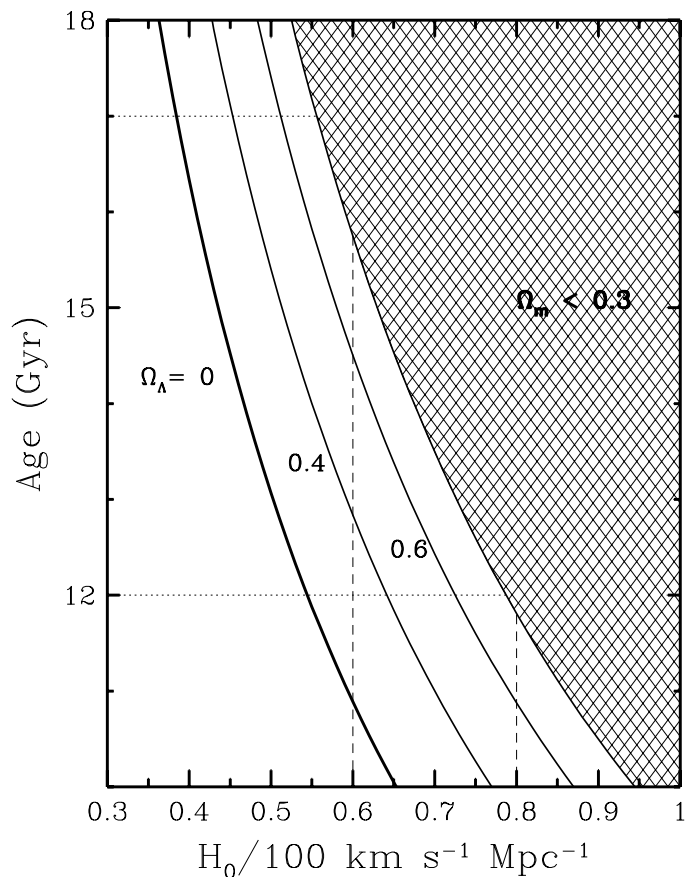
Figure 5: The relationship between age and H_0 for flat-universe models with $\Omega_{\mathrm{Matter}} = 1 - \Omega_\Lambda$. The cross-hatched region is ruled out because $\Omega_{\mathrm{Matter}} < 0.3$. The broken lines indicate the favored range for H_0 and for the age of the Universe.

secondary indicators (e.g., supernovae of Types Ia and II, infrared Tully-Fisher, and fundamental plane). For the first time in decades there is a consensus concerning the value of the Hubble constant: $H_0 = 70 \pm 10\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (where $\pm 10\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ is indicative of both the systematic and statistical errors). You don't have to be much of an optimistic to believe that a reliable determination of the local Hubble constant to a precision of 10% is within sight.

I believe that we will need to do better to really test inflation. A determination of the global Hubble constant to a precision of 5% will be needed and will require other techniques. The use of the aforementioned secondary indicators can certainly determine H_0 out to 10,000 km/s, and perhaps to 30,000 km/s. However, for a variety of CDM models (and probably any model that reproduces the observed large-scale structure) the one-sigma deviation of the local Hubble constant (within 10,000 km/s) from its global value ranges from 4% to 7% [31, 32]. Moreover, there are uncertainties associated with the secondary indicators that will be difficult to reduce below 5% (e.g., distance to LMC, Cepheid zero point, reddening

corrections for SN Ia, and so on).

The physically based methods – time delays associated with gravitational lenses, Sunyaev-Zel'dovich effect, and high-resolution mapping of CBR anisotropy – are well suited for this purpose. First, they use distant objects and thus probe the global H_0. Next, the systematics are very different and probably less subject to evolutionary and environmental effects. Finally, as we have heard at this meeting, their proponents believe that they are capable of a 5% determination. In my opinion, CBR anisotropy offers the most promise – MAP and COBRAS/SAMBA have the potential to make a one percent or better measurement of H_0.

Not having learned my lesson about speculating about the value of the Hubble constant [25], I reserve my final comments for another try: $53\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$! Let me assure the reader that the explanation is more interesting than the value. I take present measurements of the Hubble constant to be $70 \pm 10 \pm 6\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (where $\pm 6\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ reflects the one-sigma variance between the local and global values), and further, use the following prior information: age of the Universe $t_0 = 15 \pm 2\,\mathrm{Gyr}$, but necessarily greater than $10\,\mathrm{Gyr}$; big-bang cosmology is correct, which, allowing for a cosmological constant no larger than $\Omega_\Lambda = 0.7$, implies $H_0 t_0 = \frac{1}{2} - 1$. The Bayesian probability distribution for H_0 is shown in Fig. 6 – it peaks around $60\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. If I now include a prior preference for the simplest CDM models (those without nonstandard invisible matter) – which requires a smaller value of H_0, say $50 \pm 10\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (Fig. 4), and $H_0 t_0 = \frac{2}{3}$ – the probability distribution peaks around $53\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. I note that physically based measurements of the Hubble constant, which should reflect its global value, seem to be systematically smaller, though they have larger errors, and thus give some support to this value.

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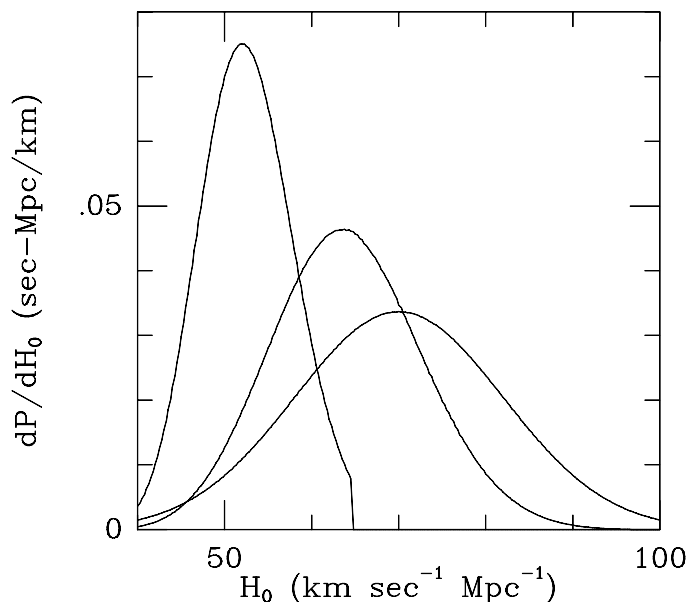[7] A.H. Guth, *Phys. Rev. D* **23**, 347 (1981); M.S. Turner, *Ann. NY Acad. Sci.* **759**, 153 (1995).

Figure 6: Probability distributions for the global Hubble constant based upon $H_0 = 70 \pm 10 \pm 6 \, \mathrm{km \, s^{-1} \, Mpc^{-1}}$ and different priors. From right to left: no priors; priors on the age and correctness of the big bang; priors on the age and correctness of the big bang and and the simplest CDM models.

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